# A depth-first implementation of Geometric Logic in Prolog

(with proof objects)

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# GL as a fragment of FOL

- Geometric formula: C => D
- $C = A_1 \wedge \ldots \wedge A_n$ ($n \geq 0$, Ai atoms)
- $D = E_1 \vee \ldots \vee E_m$ ($m \geq 0$)
- $E_j = (E\ x_1 \ldots x_k)\ C_j$ ($k \geq 0$, $C_j$ like C)
- Implicit universal closure
- No function symbols (yet), only constants

# Examples

- Lattices (meet is associative, Horn clause):
  $x \cap y = u \wedge u \cap z = v \wedge y \cap z = w \Rightarrow x \cap w = u$
- Projective unicity (resolution clause):
  $p|l \wedge p|m \wedge q|l \wedge q|m \Rightarrow p=q \vee l=m$
- Diamond property (geometric clause):
  $a \rightarrow b \wedge a \rightarrow c \Rightarrow (E\ d)\ (b \rightarrow d \wedge c \rightarrow d)$
- In general: $A_1 \wedge \ldots \wedge A_n \Rightarrow$
  $((E\ \mathbf{x})\ A_{11} \wedge \ldots \wedge A_{1i}) \vee \ldots \vee ((E\ \mathbf{y})\ A_{k1} \wedge \ldots \wedge A_{kj})$

# Rationale

- Less skolemization
- Direct proofs
- Constructive logic
- Natural proof theory/objects

# Inductive definition of X $_{(T)}$ D

- (base) X |- D if X ↓ D

- (step) $X,\underline{C_1}$ |- D, … , $X,\underline{C_n}$ |- D

  ( ) --------------------------------

  X |- D

X a finite set of facts (= closed atoms)

D closed geometric disjunction (parameters in D must occur in X)

X ↓ D iff D = ... $\vee$ (E **x**) C $\vee$ … and X contains all facts in C[**x**:=**a**] for suitable parameters **a**

( ) there exists a *closed* instance $C_0$=>$D_0$ of an axiom in T with

$C_0$ *included in* X (X contains all facts in $C_0$) and

$D_0$ = …$\vee$ (E **x**) Ci $\vee$… and each $\underline{Ci}$ a *fresh* instance of Ci ($1 \leq i \leq n$)

# Metaproperties

- Soundness
- Completeness
- Constructivity
- Conservativity
- Semidecidability
- Automation (SATCHMO!)

# Examples

- exist.in
- or.in
- nijm.in

# Case studies

- Confluence theory: induction steps in Newman's Lemma, Hindley-Rosen, Self-lengthening Thm, ..
- Lattice theory: $x \cap (y \cup z) \leq (x \cap y) \cup (x \cap z)$ for all x,y,z implies $(x \cup y) \cap (x \cup z) \leq x \cup (y \cap z)$ for all x,y,z
- Projective geometry: equivalence of two versions of Pappus' Axiom (1 minute, 1MB proof)