TYPES '04

Verification of BDD Algorithms

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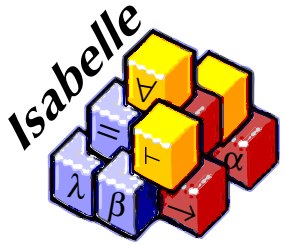

Content

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1. Subject and Goals

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2. The Verification Environment

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3. Verified BDD Algorithms

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4. Results

Subject and Goals

- Goals:
  - Verification of pointer-based data structures
  - Formal proof of the normalization algorithm on BDDs

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  - Verification of pointer-based data structures
  - Formal proof of the normalization algorithm on BDDs

- Approach:
  - Formalization of BDDs and their properties in Isabelle/Isar
  - Implementation of BDD algorithms in a C-like language
  - Specification and verification using our verification environment for Hoare Logic

Basics of the Verification Environment

- Definition of a **record** for program variables and the heap

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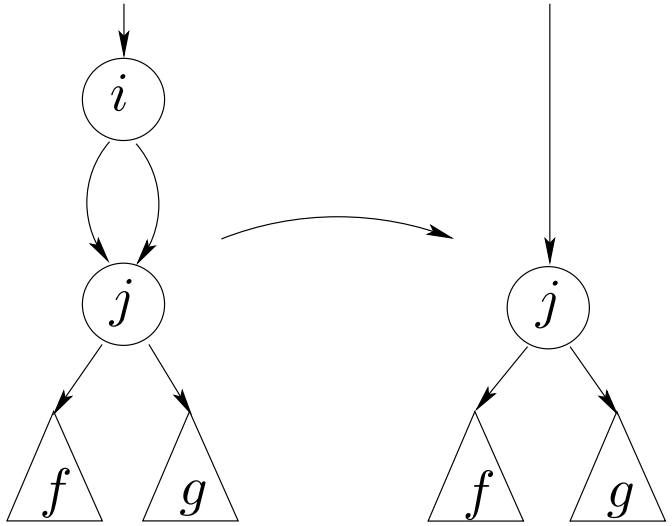
- Definition of a **record** for program variables and the heap
- Hoare-annotated program $\xrightarrow{vcg}$ proof obligation

  This proof obligation is the basis for our verification work.

Properties of BDDs

reduced:



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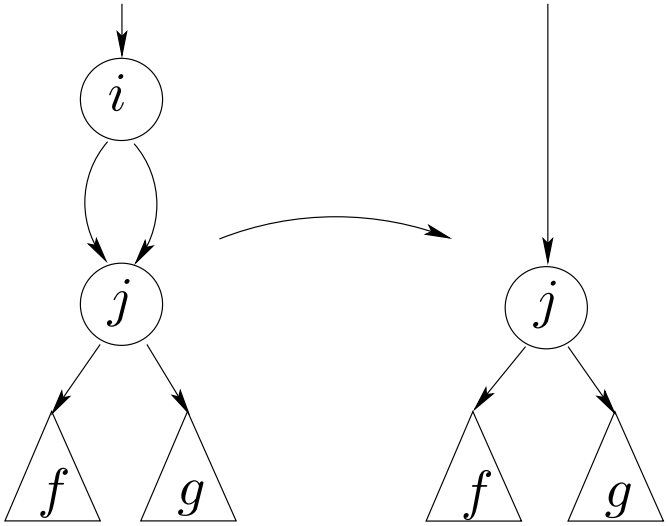

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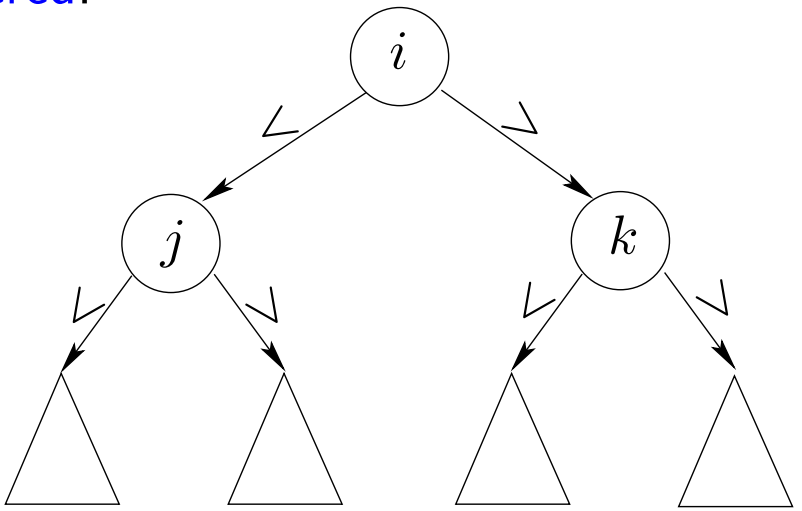

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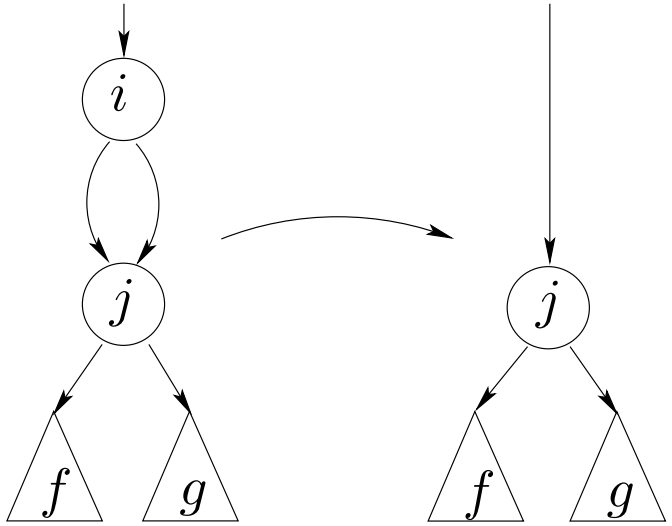

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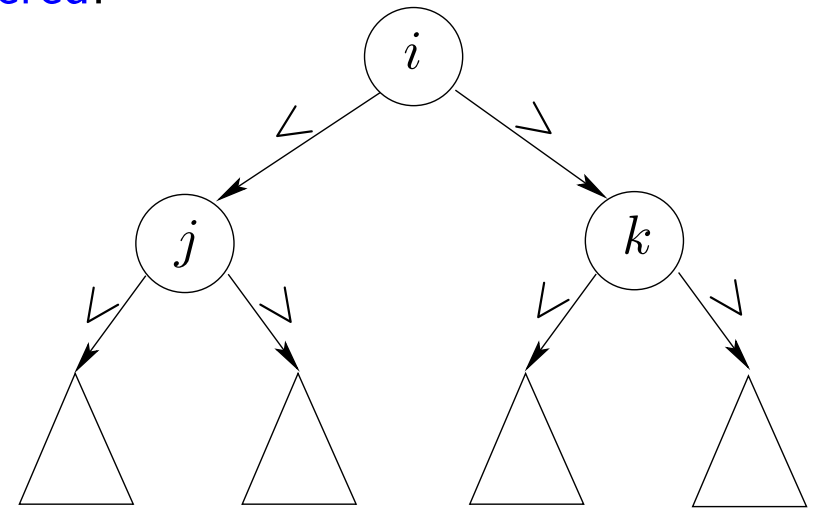
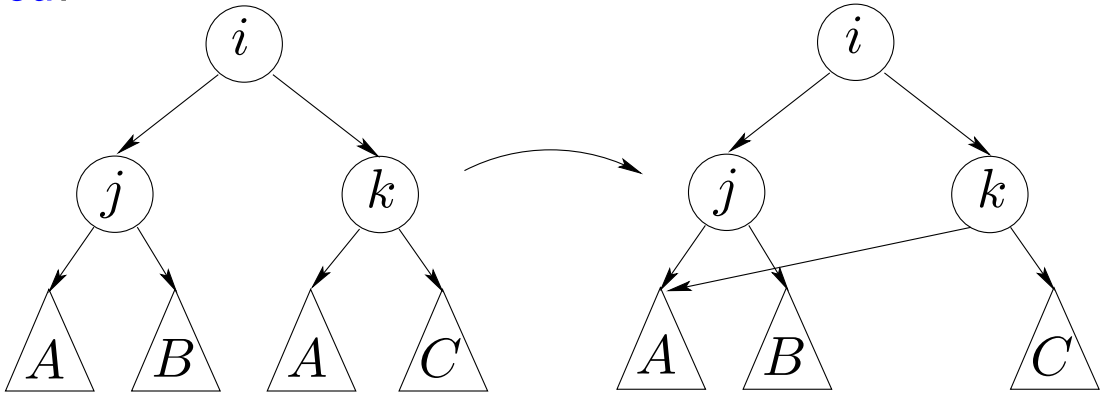


shared:



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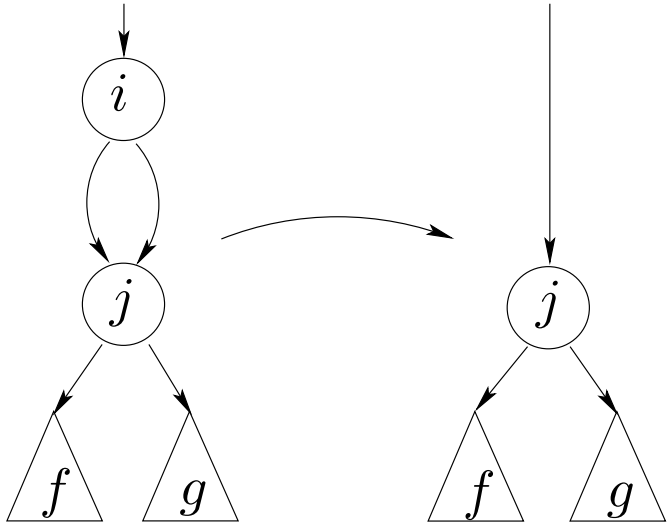

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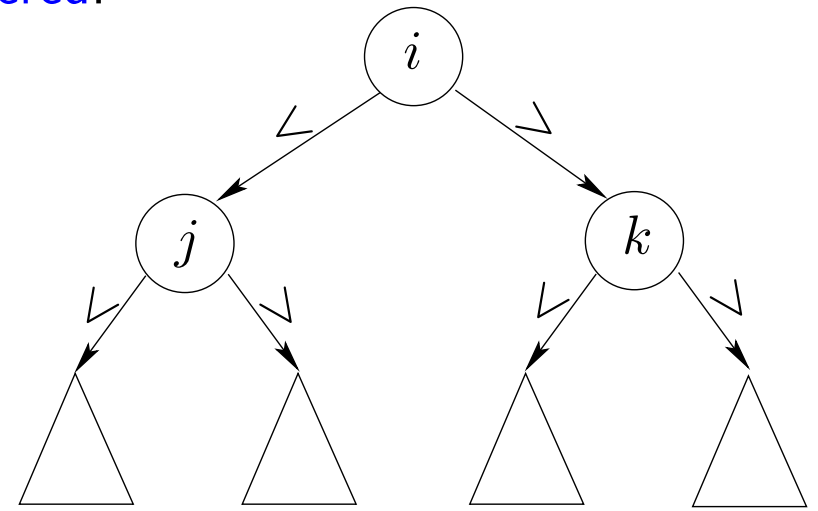

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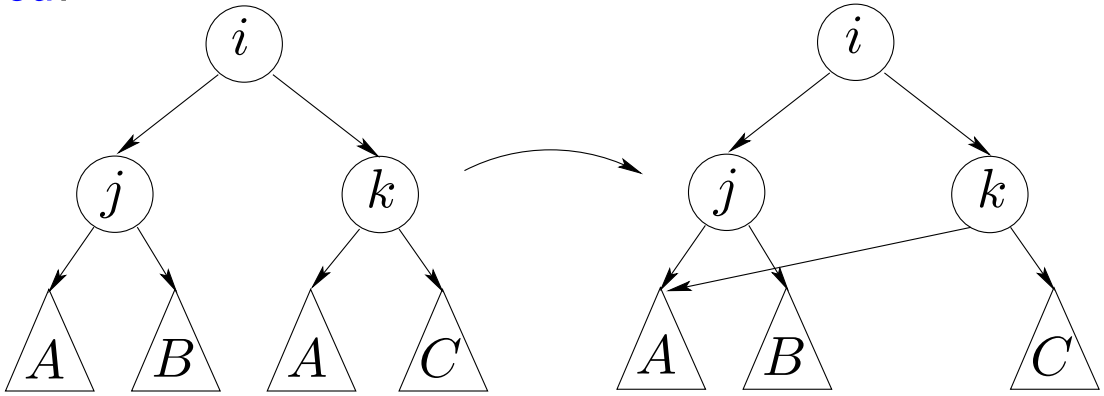
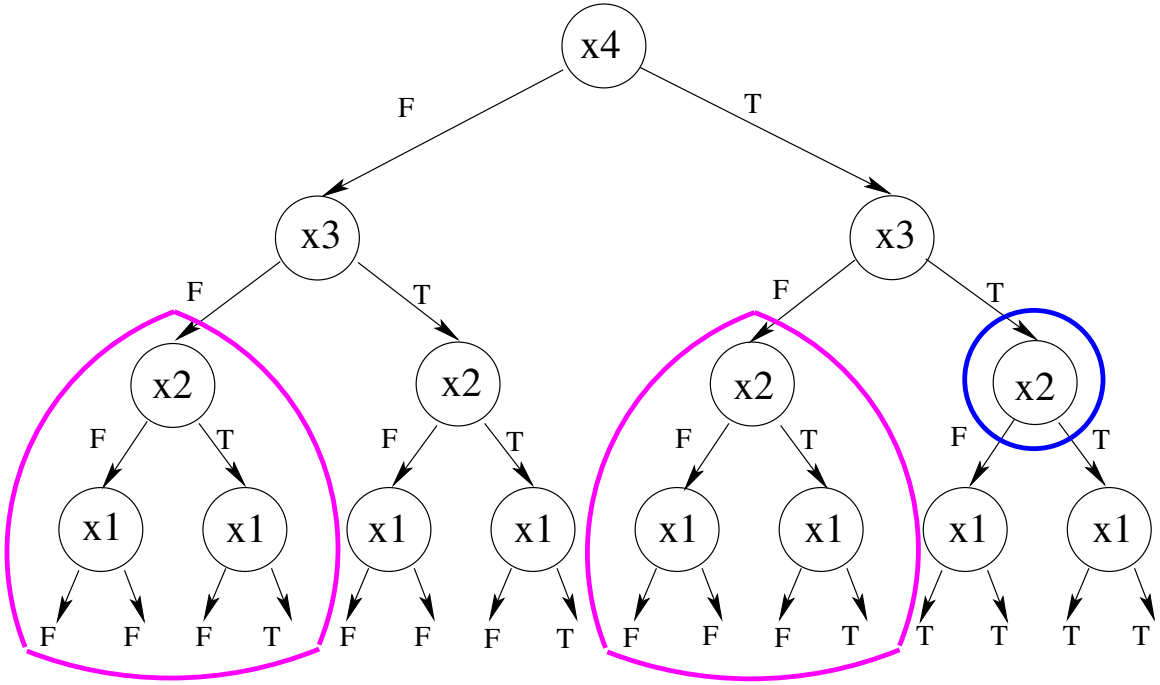


Result:

- Unique representation for each function
- No redundant parts

Example

$$f(x1, x2, x3, x4) := x1 \wedge x2 \vee x3 \wedge x4$$



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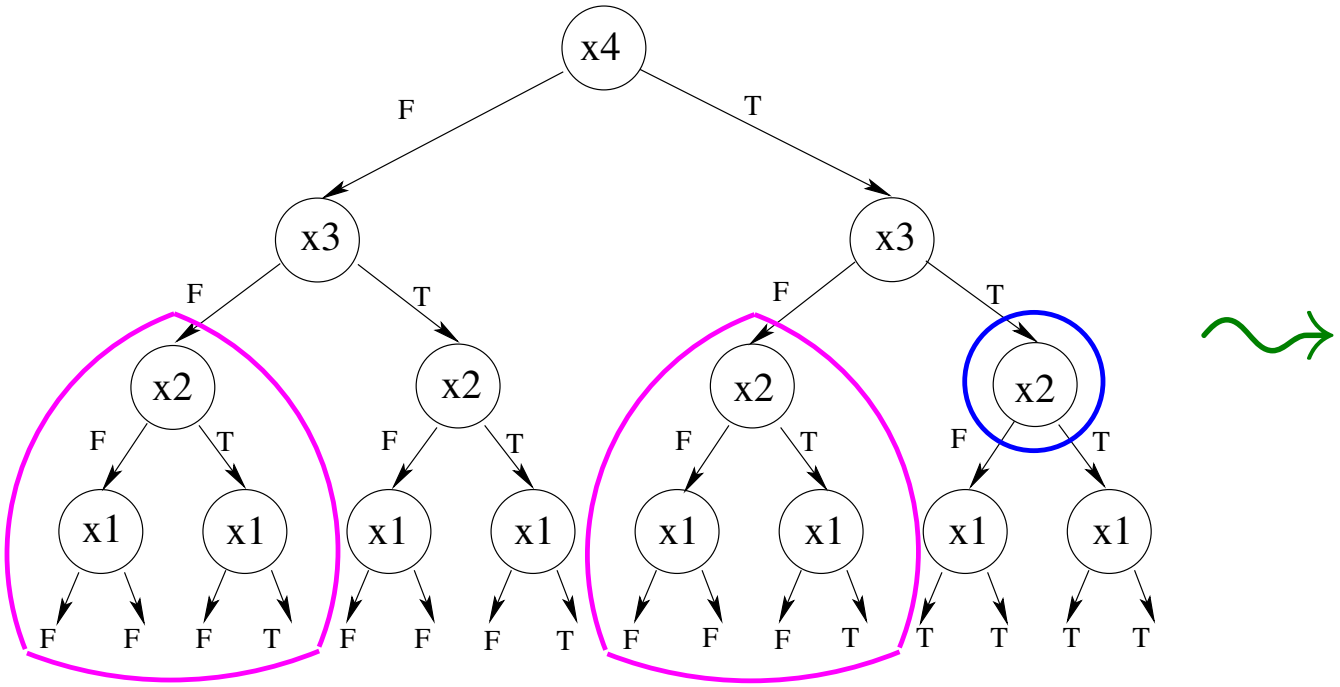

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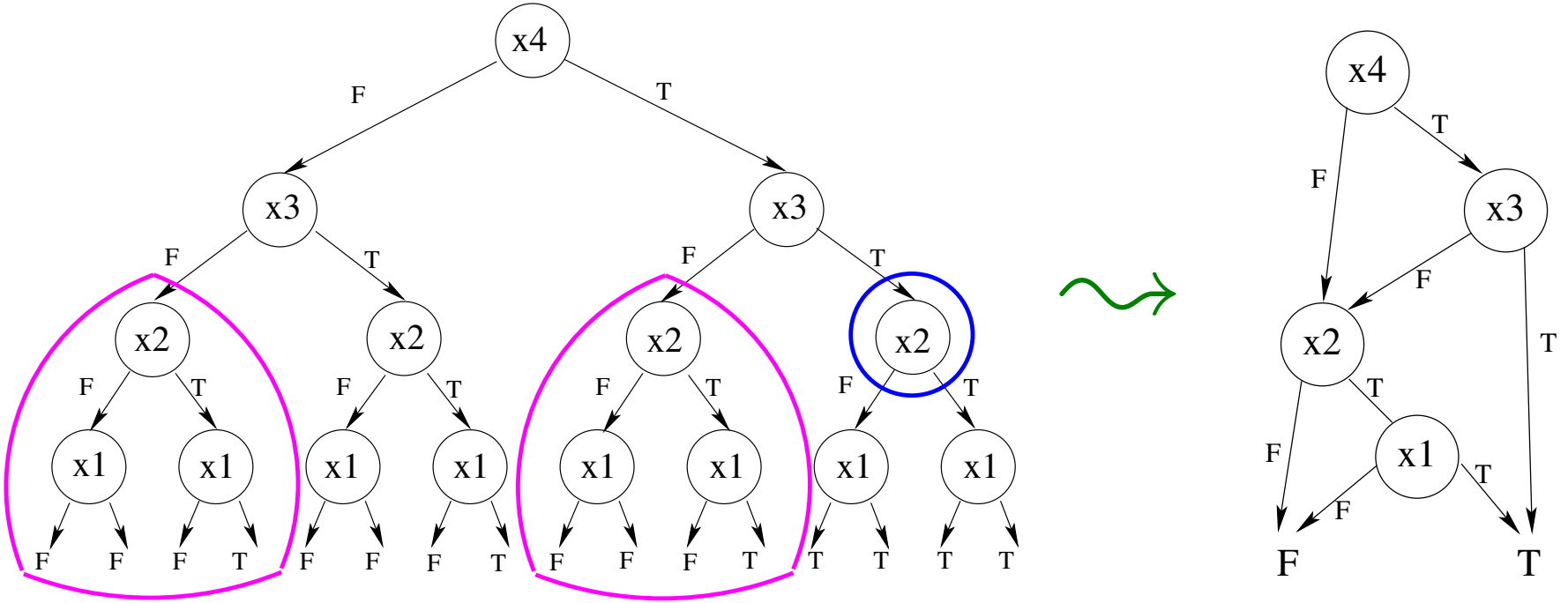
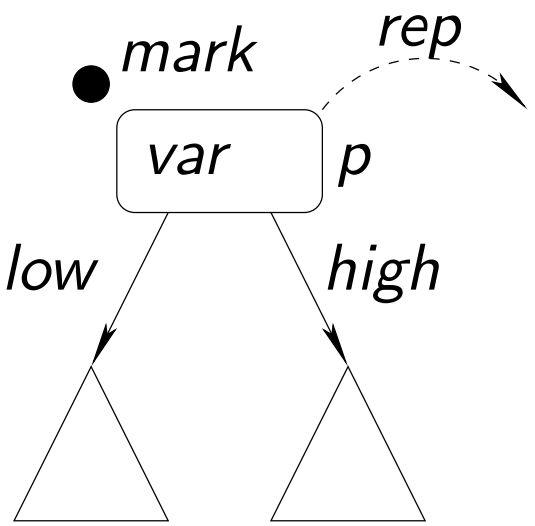


Representation of BDDs in the heap

```
struct {
   nat var;
   node* low;
   node* high;
   node* rep;
   bool mark;
} node;
```



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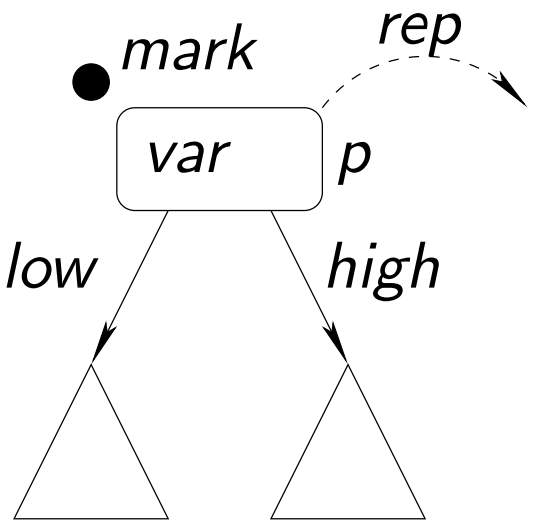

record *bdd_data* =
  var :: *ref* ⇒ *nat*
  low :: *ref* ⇒ *ref*
  high :: *ref* ⇒ *ref*
  ref :: *ref* ⇒ *ref*
  mark :: *ref* ⇒ *bool*
  ⋮

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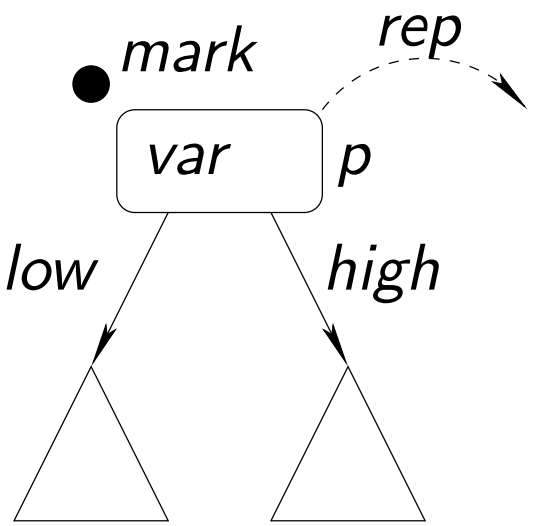

record $bdd\_data =$
$var :: ref \Rightarrow nat$
$low :: ref \Rightarrow ref$
$high :: ref \Rightarrow ref$
$ref :: ref \Rightarrow ref$
$mark :: ref \Rightarrow bool$
$\vdots$

bdd_data shows split heap approach
Advantage:
Specification of properties on relevant heap components instead of on whole heap

BDD Formalization - Illustration

2 levels of specification:

- Graph structure in order to represent *sharing*:
  datatype $dag = Tip \mid Node\ dag\ ref\ dag$

- Decision tree (e.g. for evaluation purposes):
  datatype $bdt = Zero \mid One \mid Bdt\_Node\ bdt\ nat\ bdt$

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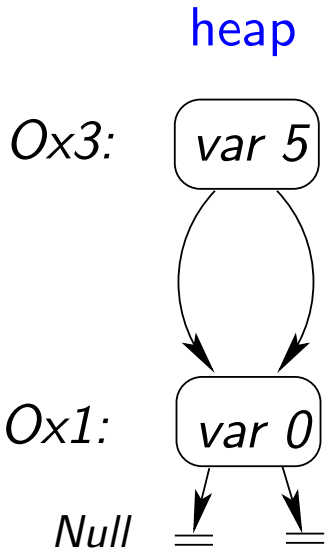

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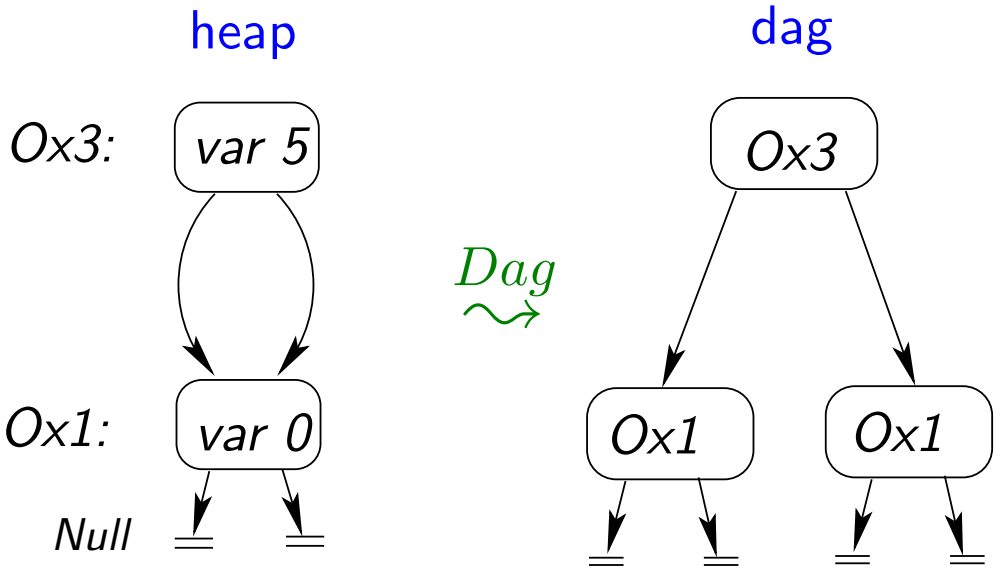

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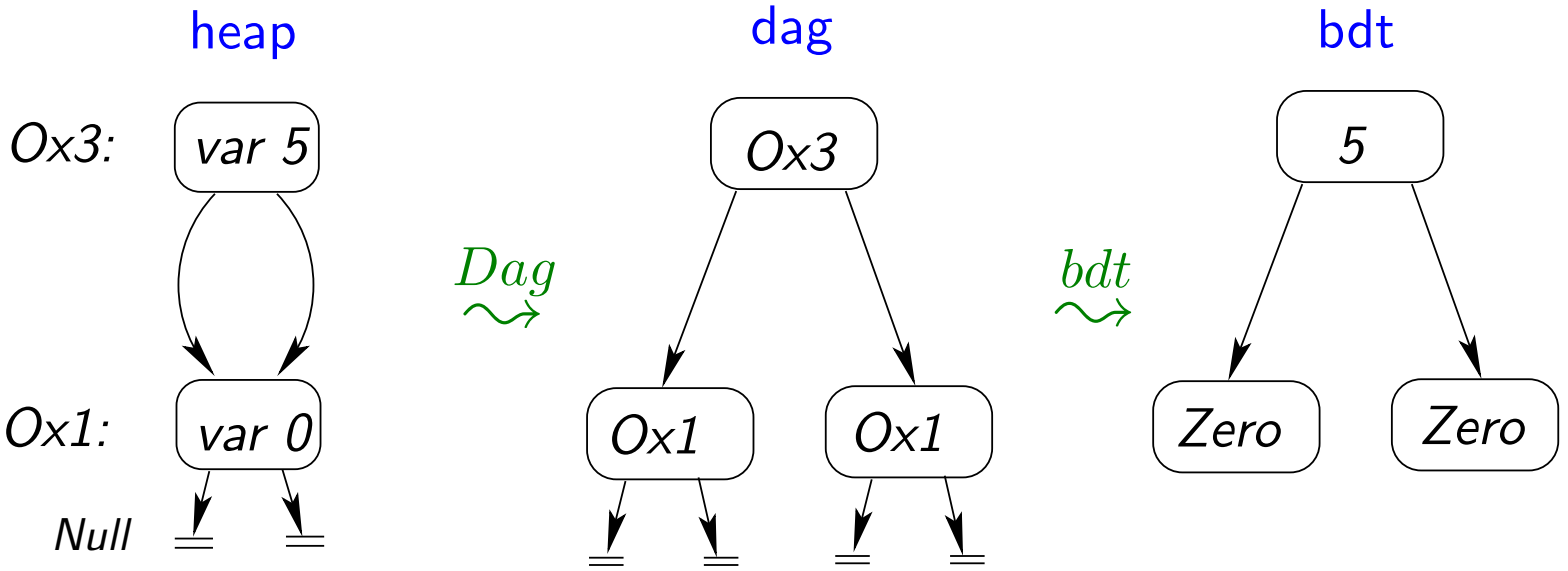

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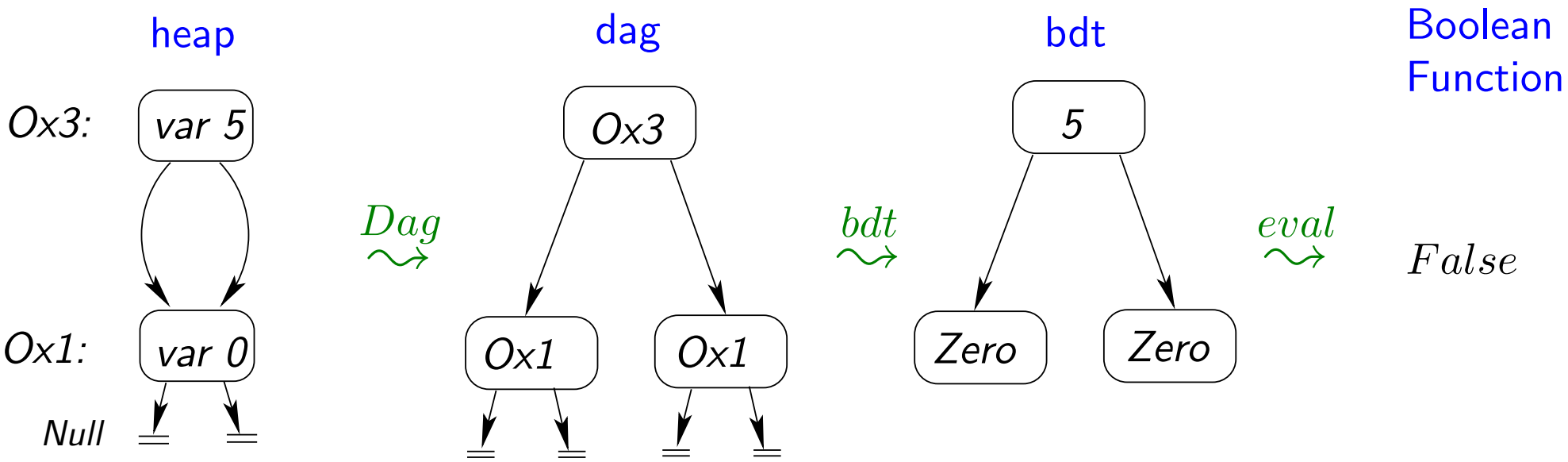

Transition Functions

- Construction of a dag in the heap:
  consts $Dag :: ref \Rightarrow (ref \Rightarrow ref) \Rightarrow (ref \Rightarrow ref) \Rightarrow dag \Rightarrow bool$
  primrec
  $Dag\ p\ low\ high\ Tip = (p = Null)$
  $Dag\ p\ low\ high\ (Node\ ldag\ a\ rdag) = (p = a \wedge p \neq Null \wedge$
  $Dag\ (low\ p)\ low\ high\ ldag \wedge Dag\ (high\ p)\ low\ high\ rdag)$

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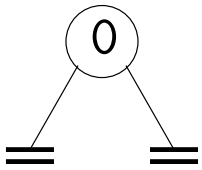
- Construction of a bdt out of a dag:
  consts $bdt :: dag \Rightarrow (ref \Rightarrow nat) \Rightarrow bdt\ option$

Problems with the bdt Conversion

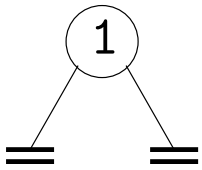
Reasons, why the conversion could go wrong:

- $bdt\ Tip\ var = ?$
- bdt-endnodes ($Zero/One$) must be represented by an inner node in the dag-representation:

  $Zero$ node:

  One node:

- No inner bdt-node may contain variable values 0 and 1
- Balanced structure of bdt-nodes

Normalize - Auxiliary Functions for the Specification

- eval:
  consts $eval :: bdt \Rightarrow (nat \Rightarrow bool) \Rightarrow bool$

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- isomorphic_dags_eq:
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  $\forall bdt_1\ bdt_2.\ (bdt\ dag_1\ var = Some\ bdt_1 \wedge bdt\ dag_2\ var = Some\ bdt_2 \wedge$
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- shared:
  $shared\ dag\ var \equiv$
  $\forall dag_1\ dag_2.\ (dag_1 \leq dag \wedge dag_2 \leq dag) \longrightarrow isomorphic\_dags\_eq\ dag_1\ dag_2\ var$

Normalize - Specification

```
‘levellist := replicate (‘p→‘var + 1) [];
‘levellist := CALL Levellist(‘p, ¬‘p→‘mark, ‘levellist);
‘n := 0;
WHILE ‘n < length ‘levellist
DO CALL ShareReduceRepList(‘levellist !  ‘n); ‘n := ‘n + 1 OD;
‘p := CALL Repoint(‘p)
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Normalize - Specification

$\forall\ \sigma\ predag\ prebdt.\ \Gamma \vdash$
$\{\!|\sigma.\ Dag\ `p\ `low\ `high\ predag \wedge ordered\ predag\ `var \wedge$
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$\{\!|\ (\exists\ postdag.\ Dag\ `p\ `low\ `high\ postdag \wedge reduced\ postdag \wedge shared\ postdag\ ^{\sigma}var \wedge$
$\quad ordered\ postdag\ ^{\sigma}var \wedge set\_of\ postdag \subseteq set\_of\ predag \wedge$
$\quad (\exists\ postbdt.\ bdt\ postdag\ ^{\sigma}var = Some\ postbdt \wedge prebdt \sim postbdt))|\!\}$

Results

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- Complete formalization of the BDD concept
- First machine-checked proof of the normalization algorithms
- Verification of C-like pointer programs possible using the verification environment for Hoare Logic in Isabelle

Thank you for your attention!