

Types 2004
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A minimalist FOUNDATION
FOR
CONSTRUCTIVE MATHEMATICS

M. Mzietti & G. Sambin

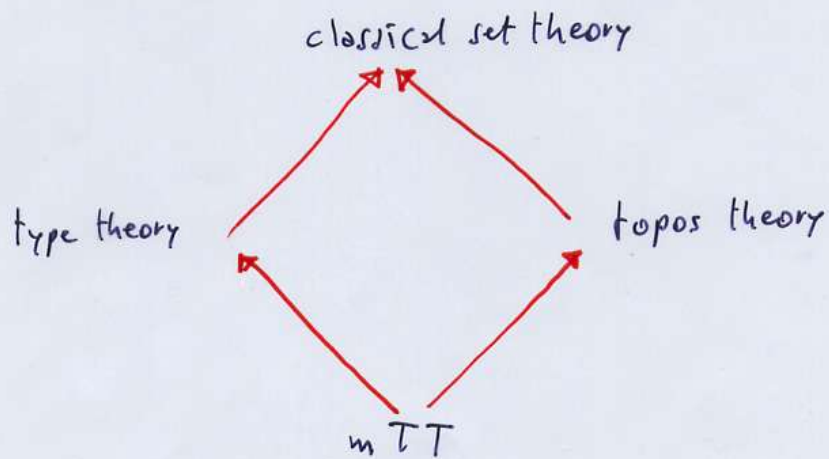
T satisfies proofs-as-programs
 $\Rightarrow T$ consistent with $AC+CT$

(\pm Ext) $AC: \forall x \exists y R(x,y) \rightarrow \exists f \forall x R(x,fx)$

$CT: \forall f: \mathbb{N} \rightarrow \mathbb{N} \exists e \forall n (T(e,n,z) \wedge z=fn)$

but $\left. \begin{array}{l} ZF \\ \text{ML84} \\ \text{topos th.} \end{array} \right\} + AC+CT \vdash \perp$

ExtPow + AC \vdash EM



abandon propositions-as-sets

\exists via the reflection principle

$$\Gamma, (\exists x \in D) A(x) \vdash \Delta \quad \text{iff} \quad \text{for every } d \in D, \\ \Gamma, A(d) \vdash \Delta$$

$$\Gamma, (\exists x \in D) A(x) \vdash \Delta \quad \text{iff} \quad \Gamma, z \in D, A(z) \vdash \Delta \\ z \text{ not free in } \Gamma, \Delta$$

$$\exists\text{-formation} \quad \frac{\Gamma, z \in D, A(z) \vdash \Delta}{\Gamma, (\exists x \in D) A(x) \vdash \Delta}$$

\exists -implicit reflection

$$\frac{\Gamma, (\exists x \in D) A(x) \vdash \Delta}{\Gamma, z \in D, A(z) \vdash \Delta} \quad \text{compose cut } (\exists x \in D) A(x)$$

trivialize $\Gamma = \emptyset$
 $\Delta = (\exists x \in D) A(x)$

\exists -axiom $z \in D, A(z) \vdash (\exists x \in D) A(x)$

$$\frac{\Gamma \vdash z \in D \quad \Gamma' \vdash A(z)}{\Gamma, \Gamma' \vdash (\exists x \in D) A(x)}$$

compose cut $z \in D$
 $A(z)$

trivialize $\Gamma = z \in D$
 $\Gamma' = A(z)$

\exists -explicit reflection

minimal Type Theory mTT

sets: exactly as in Martin-Löf

Σ -elimination

$$\frac{\begin{array}{l} [z \in (\Sigma_{x \in B} C(x))] \\ M(z) \text{ set} \end{array} \quad \begin{array}{l} d \in (\Sigma_{x \in B} C(x)) \\ m_{(x,y)} \in M(\langle x,y \rangle) \end{array} \quad \begin{array}{l} [x \in B, y \in C(x)] \\ m_{(x,y)} \in M(\langle x,y \rangle) \end{array}}{\Sigma \text{ elim}(d, m) \in M(d)}$$

propositions: intuitionistic logic, A prop

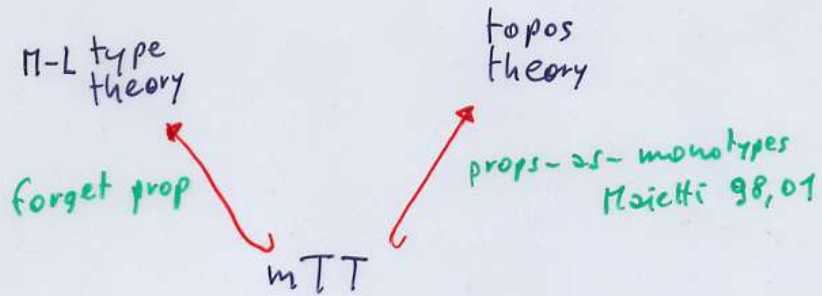
\exists -elimination

$$\frac{\begin{array}{l} M \text{ prop} \\ d \in (\exists x \in B) C(x) \end{array} \quad \begin{array}{l} m_{(x,y)} \in M \end{array} \quad \begin{array}{l} [x \in B, y \in C(x)] \\ m_{(x,y)} \in M \end{array}}{\exists \text{ elim}(d, m) \in M}$$

link: propositions-into-sets

$$\frac{A \text{ prop}}{A \text{ set}}$$

Compatibility:



toolbox: that for M-L type th.

normalization: that for M-L type th.

type checking: open, but seems clear

realizability interpretation: open, but have an idea

other literature: Howard 69-79
Mitchell-Plotkin ?
Aczel-Gambino 02
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