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An E-bicategory of E-categories

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Slogan: Objects are not equal, but isomorphic

- E-categories support this idea
- Leads to higher-dimensional category theory
- Can't have the category of categories



Using Agda:

- Formalized bicategories
- Formalized the bicategory of categories

- No inductive families



Recall: A bicategory B consists of

- A collection of objects
 - For every pair a, b of objects, a hom-category $B(a, b)$
 - Composition functors $B(b, c) \times B(a, b) \longrightarrow B(a, c)$
 - associative up to given natural isomorphism
 - Identity functors $1 \longrightarrow B(a, a)$
 - with given “unit morphisms”
- and satisfies some axioms.



Problems when formalizing:

- With a little help, things easily get out of hand.
 - A failed attempt.
 - Close attention needed to all details
- Very long formalization
 - Hidden arguments may help



An oddity:

```
RevElimLeftIdfunctorNatTra (C::E_category)(D::E_category)(F::E_functor C D)
  :: E_natural_transformation C D
   F (E_functorcomposition C D D (E_idfunctor D) F)
= struct
  arrows      = (ElimLeftIdfunctorNatTra C D F).arrows
  ax_naturality = (ElimLeftIdfunctorNatTra C D F).ax_naturality
```

works, but

```
RevElimLeftIdfunctorNatTra (C::E_category)(D::E_category)(F::E_functor C D)
  :: E_natural_transformation C D
   F (E_functorcomposition C D D (E_idfunctor D) F)
= ElimLeftIdfunctorNatTra C D F
```

gives an error.